Unit III : 2D and 3D Transformations

3.1 2D and 3D Transformations

Transformation is the process of modifying and repositioning an object in a two-dimensional (2D) or three-dimensional (3D) space. Common transformations include:

- Translation
- Rotation
- Scalling
- Reflection
- Shearing

> → Transformation can be applied on already created object.
> → Transformation plays an important role to reposition the graphics on the screen and change their size or orientation

Each transformation can be represented using transformation matrices, which are applied to points or objects to achieve the desired effect.

2D Transformation :- 2D transmissions are operations applied to geometric objects to change their position, size, or orientation in a two-dimensional plane.

1. Translation

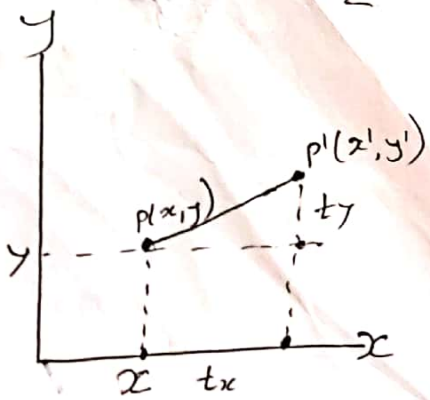
Translation moves a point or an object from one position to another by adding a fixed values to its coordinates.

formula : If a point $P(x, y)$ is translated by tx in the x-direction and ty in the y-direction, the new point $P'(x', y')$ is:

$$x' = x + tx$$
$$y' = y + ty$$

Matrix Representation:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



P(x, y) is represented as:

$$\begin{bmatrix} x \\ y \end{bmatrix}$$

Translation matrix represented as:

$$T = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix}$$

To find the new coordinates $p'(x'y')$, we multiply the transformation matrix T with the point matrix:

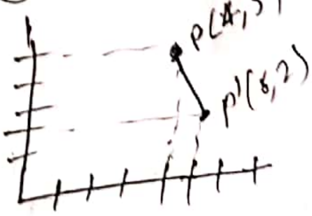
$$P' = P.T$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \end{bmatrix}$$

$$p' = p + T$$

Example: Translate co-ordinate point $(\overset{x}{4}, \overset{y}{5})$ by t_x, t_y $(2, -3)$.



Solution

$x' = x + t_x$
$= 4 + 2$
$= 6$

$y' = y + t_y$
$= 5 + (-3)$
$= 2$

(6, 2) is the translate new coordinate points

We can apply translation on:

- Point
- Line
- Rectangle

Example of Translation

Consider coordinates (4, 6), find the new coordinates without changing the radius, apply the translation technique with distance 6 towards x-axis and 4 towards y-axis.

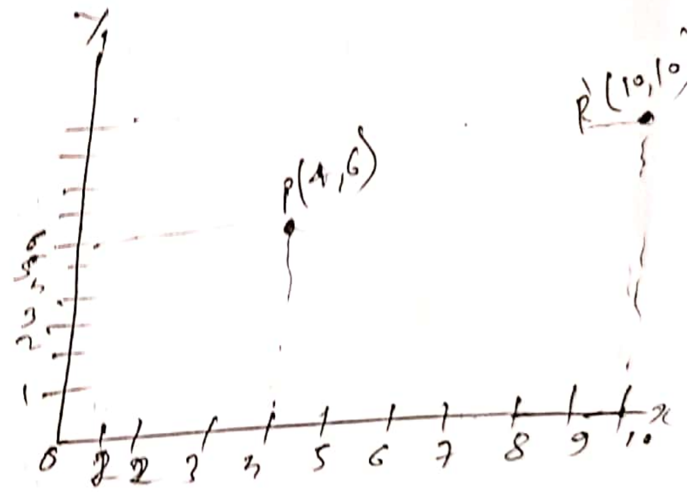
Solution

Given $P(\overset{x}{4}, \overset{y}{6})$

$Tx = 6$

$Ty = 4$

$P'(x', y') = ?$



$x' = x + Tx$

$= 4 + 6$

$= 10$

$y' = y + Ty$

$= 6 + 4$

$= 10$

Hence

$P'(x', y') = (10, 10)$

Trigonometry table

	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Rotation

Rotation moves a point or object around a fixed point, usually the origin, by an angle θ.

2D Rotation About origin:

$$x' = x\cos\theta - y\sin\theta$$

$$y' = x\sin\theta + y\cos\theta$$

$+90$ Anticlockwise

Rotating point $(x, y) = (-y, x)$

Clockwise - rightward from the top

Anticlockwise (Counter clockwise) → leftward from the top.

If θ is positive, the rotation is Anticlockwise

If θ is negative, the rotation is Clockwise.

Rotating a point (x, y) Clockwise $= (y, -x)$

Example Rotate $(2, 3)$ by 90° counterclockwise

$x' = x\cos 90^\circ - y\sin 90^\circ$
$= 2 \times 0 - 3 * 1$
$= -3$

$(-3, 2)$

$y' = x\sin 90^\circ + y\cos 90^\circ$
$= 2 \times 1 + 3 \times 0$
$= 2$

2D Scaling :-

Scaling concept is used to alters the size of an object. In this process, we can either expand or reduce the dimension of the object.

Scaling operation can be done by multiplying every vertex coordinate (x, y) of the polygon with scaling factor Sx and Sy to produce the transformed coordinates as (x', y').

So,
$$x' = x * Sx$$
$$y' = y * Sy.$$

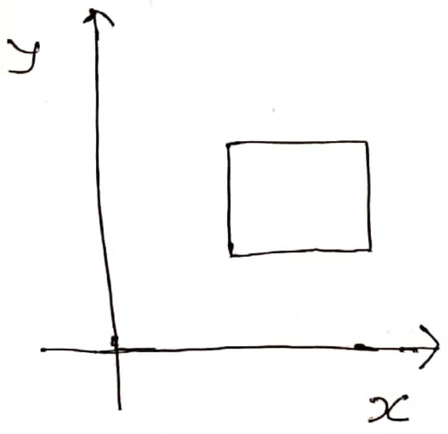
Where Sx, Sy are scaling factors which scales the object in x and y direction. The matrix representation of scalling is:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

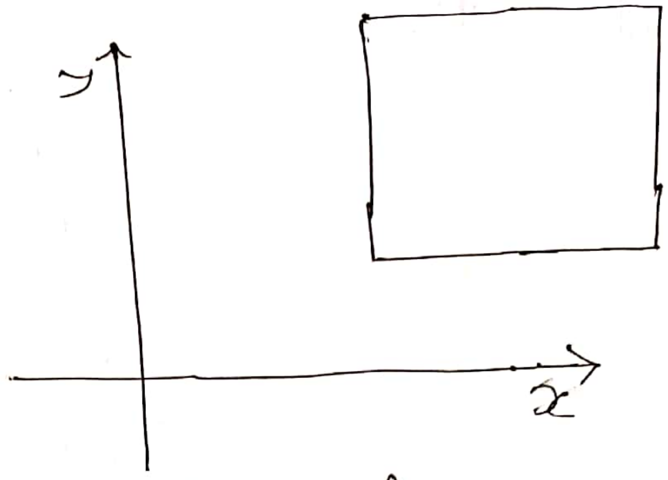
Resulting in:

$$x' = Sx . x$$
$$y' = Sy . y$$

OR $P' = S.P$



Before scaling
$P(x, y)$



After scaling
$P'(x', y')$

Note: Suppose the scaling factor is less than 1, reduce the size of the object, greater than 1, increase size of the object, equal to 1 then new scaling object $P'(x', y')$ is same as $P(x, y)$ No change in the object size.

Points to remember :

↳ If scaling factor is one, that is $s_x = 1$ and $s_y = 1$ then there is no change in object size.

$x' = x . s_x = x$
$y' = y . s_y = y$

↳ If scaling factor is greater than one, then the object size is enlarged.

↳ If scaling factor is less than one, then the object size is reduced.

Example of Scaling

Increase the size of object to double, that is Scaling factor is 2 in x and y axis. Let the coordinate points A(1,4), B(4,4), C(4,1), D(1,1). Now find the new coordinates of object?

Solution

Given

A(1,4)

B(4,4)

C(4,1)

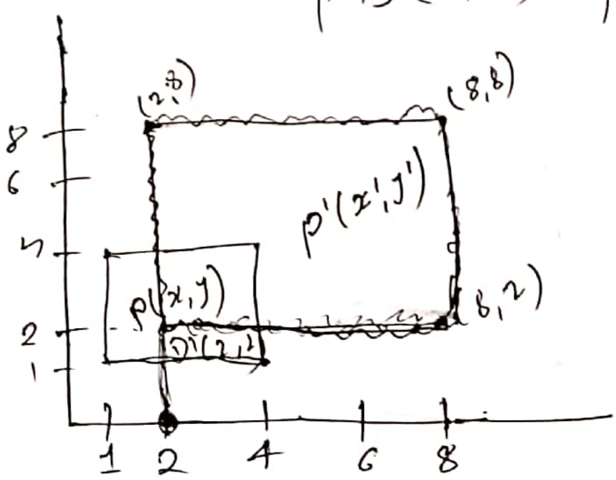
D(1,1)

$Sx = 2$

$Sy = 2$

Now

A(1,4)	B(4,4)	C(4,1)	D(1,1)
$x' = Sx.x$	$x' = Sx.x$	$x' = Sx.x$	$x' = Sx * x$
$= 2*1$	$= 2\times4$	$= 2\times4$	$= 2\times1$
$= 2$	$= 8$	$= 8$	$= 2$
$y' = Sy.y$	$y' = Sy*y$	$y' = Sy.y$	$y' = Sy*y$
$= 2*4$	$= 2*4$	$= 2\times1$	$= 2*1$
$= 8$	$= 8$	$= 2$	$= 2$
A'(2,8)	B'(8,8)	C'(8,2)	D'(2,2)



Reflection:- 2D reflection evaluates the mirror image of an object.

Reflection can be done!

- ↳ Along x-axis
- ↳ Along y-axis
- ↳ About $x = y$ line.
- ↳ Along origin

✗ Along x-axis.

Consider a point $p(x, y)$ on x-y plane then $P'(x', y')$ is the reflection about x-axis is given as

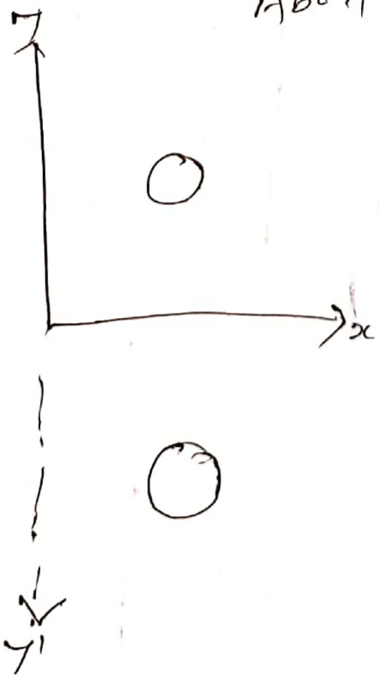
$$x' = x \quad \text{and} \quad y' = -y$$

Matrix form Representation:

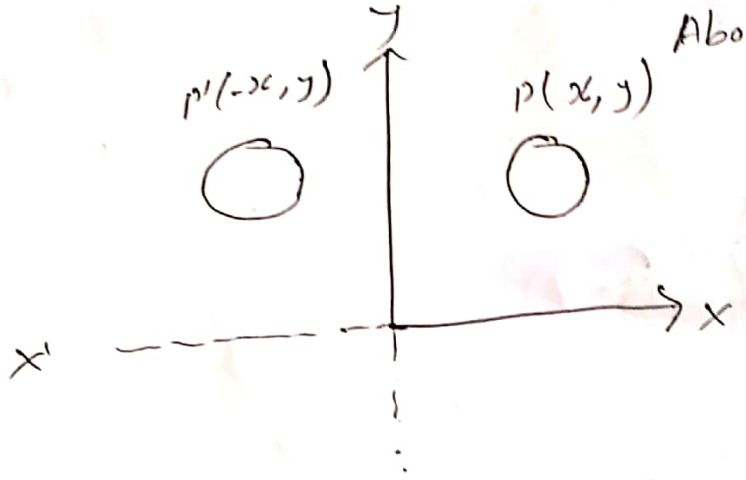
$$\begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

$$P' = P \cdot R_x$$

About X-axis



About y-axis



$x' = -x$

$y' = y$

* Matrix form along y-axis:

$$\cancel{\begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \cdot \begin{bmatrix} R_0 \end{bmatrix} \cdot \begin{bmatrix} R_y \end{bmatrix} \cdot \begin{bmatrix} R_0 \end{bmatrix}}$$

$$\begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

$$P' = P \cdot R_y$$

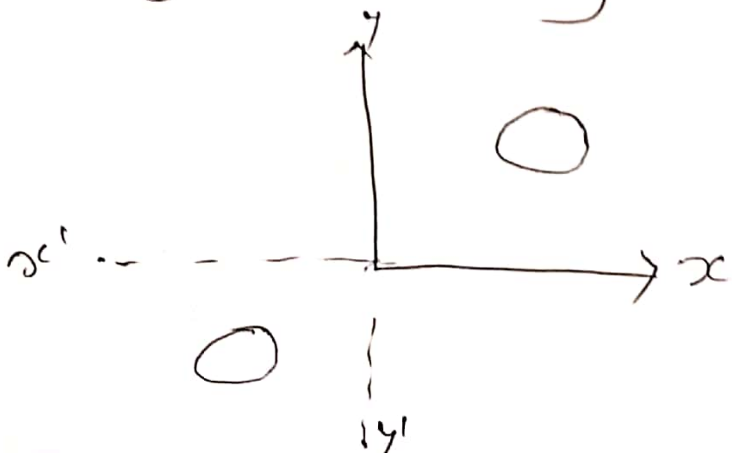
*About $x = y$ line $p(x, y)$ then $p'(x', y')$

$x' = y$

$y' = x$

Matrix form:

$$\begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

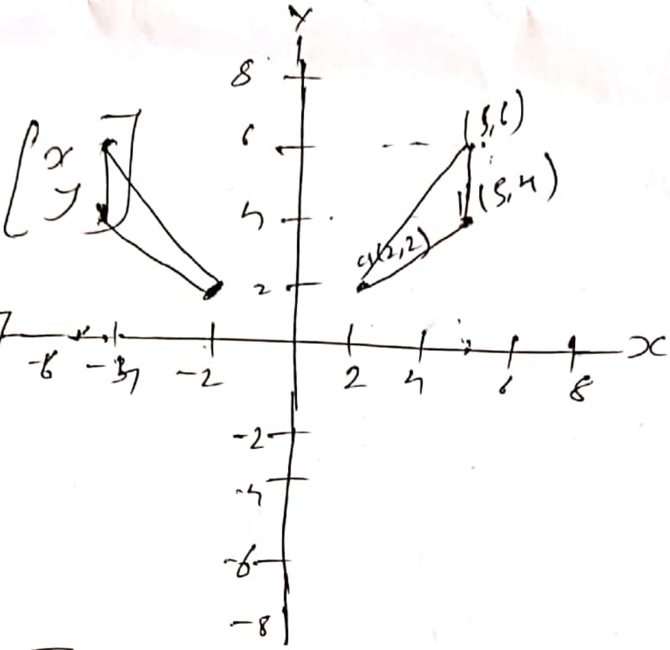


Example of Reflection

A triangle with the coordinates $p(5,4)$, $q(2,2)$, $r(5,6)$, now we need to reflect it along y-axis

Solution

$p(5,4)$ $\quad Re(y)$
$\quad\; x_0 \;\; y_0$



$$p' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} -5 \\ 4 \end{bmatrix}$$

$$q' = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2+0 \\ 0+2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$r' = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} -5+0 \\ 0+6 \end{bmatrix}$$

$$= \begin{bmatrix} -5 \\ 6 \end{bmatrix}$$

2D Shearing :-

Distorts - [illegible]
Slant - झुकाव, वाकडा

A transformation that distorts the shape of an object such that the transformed shape appears as if it has been "slanted" in either the x-direction, y-direction, or both.
It shifts the object's points along an axis in porportion to their distance from the other axis.

Types of shearing in 2D

1. X direction shearing :- Shifts points in the x-direction, keeping the y-cordinates unchanged
2. y-direction Shearing :- Shifts points in the y-direction, keeping the x-coordinates unchanged.
2. xy-direction Shearing :- Shifts points in both x and y coordinates simultaneously.

1) X-direction shearing

$$x' = x + sh_x * y$$
$$y' = y$$

Matrix Representation :-

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & sh_x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Example :

Point (2,3) with shearing factor $sh_x = 2$.

$$x' = x + sh_x . y = 2 + 2*3 = 8$$

$$y' = y = 3$$

New points After shearing is (8,3) Ans

2. Y-Direction shearing

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ sh_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x' = x$$
$$y' = y + sh_y * x$$

Example:-

A point (2,3) with shearing factor $sh_y = 1.5$

$x' = 2$

$$\begin{aligned} y' &= y + sh_y * x \\ &= 3 + 1.5 * 2 \\ &= 6 \end{aligned}$$

Hence,
New points after y-shearing is (2, 6) Ans

3. Xy-Direction shearing

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & sh_x \\ sh_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x' = x + sh_x * y$$
$$y' = y + sh_x * y$$

Example: A point (2, 3) with $sh_x = 2$ and $sh_y = 1.5$

$$\begin{aligned} x' &= x + sh_x * y \\ &= 2 + 2 * 3 \\ &= 2 + 6 \\ &= 8 \end{aligned}$$

$$\begin{aligned} y' &= y + sh_y * x \\ &= 3 + 1.5 * 2 \\ &= 3 + 3 \\ &= 6 \end{aligned}$$

New point After shearing x & y is (8, 6) Ans

Representation of 2D and 3D Transformation in Homogeneous Coordinate System

In computer graphics, transformations like translation, scaling, rotation, and shearing are commonly used to manipulate objects. Using homogeneous coordinates, we can represent all transformations in matrix form, making computations easier.

2. Homogeneous Coordinates

In 2D, a point $p(x, y)$ is represented as $p(x, y, 1)$
In 3D, a point $p(x, y, z)$ is represented as $p(x, y, z, 1)$.

2D Transformations in Homogeneous Coordinates

A 2D point (x, y) is written in homogeneous form as $(x, y, 1)$.

1. Translation (Moving the object): Moves a point by T_x and T_y.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

formula:

$$x' = x + T_x$$
$$y' = y + T_y$$

2. Rotation (Rotating the object by θ)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

formula:

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta - y\cos\theta$$

3. Scaling (Resizing the object by S_x and S_y factors).

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

formula,

$$x' = S_x * x$$

$$y' = S_y * y$$

4. Shearing (slanting the object)

1. x-Shear:

$$\begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = x + sh_x * y$$

$$y' = y$$

2. y-Shear

$$\begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = x$$

$$y' = y + sh_y * x$$

3D Homogeneous Transformation with formula

In 3D computer graphics, transformations such as translation, rotation, scaling and shearing are commonly applied to objects.

In homogeneous coordinates, a 3D point $p(x, y, z)$ is representend as:

$$p(x, y, z) \rightarrow P_h(x, y, z, 1)$$

3D Translation (Moving the object)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

formula,

$$x' = x + T_x, \quad y' = y + T_y, \quad z' = z + T_z$$

3D Rotation (Rotating the object)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

formula * About x-axis

$$y' = y\cos\theta - z\sin\theta$$

$$z' = y\sin\theta + z\cos\theta$$

* About y-axis Rotation

$$x' = x\cos\theta + z\sin\theta$$

$$z' = -x\sin\theta + z\cos\theta$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

* About z-axis (z-axis)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$x' = x\cos\theta - y\sin\theta$$

$$\cancel{y' = x\sin\theta + y\cos\theta}$$

$$y' = x\sin\theta + y\cos\theta$$

3. 3D scalling (Resizing the object)

Scales a point by S_x, S_y, and S_z:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

formula,

$$x' = S_x * x$$
$$y' = S_y * y$$
$$z' = S_z * z$$

4. 3D Shearing (Slanting the object): Shearing changes the shape by shifting one coordinate proportinally to another.

(i) Shearing in x-Direction

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_{xy} & sh_{xz} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$x' = x + sh_{xy} * y + sh_{xz} * z$$

Q. A point $P(2,3,4)$ is translated by $(3,-2,5)$ and then rotated 30° about the x-axis find the new coordinates.

$x' = x + t_x$	$y' = y + t_y$	$z' = z + t_z$
$= 2+3$	$= 3+(-2)$	$= 4+5$
$= 5$	$= 1$	$= 9$

Now

Rotate x-Axis

$y' = y\cos\theta - z\sin\theta$

$z' = y\sin\theta + z\cos\theta$

$y' = 3\cos 30 - 4\sin 30$

$= 3 \times \frac{\sqrt{3}}{2} - \cancel{4}^{2} \times \frac{1}{\cancel{2}}$

$= 2.59 - 2 = 0.59$

$z' = y \sin 30° + z \cos 30°$

$= 3 \times \frac{1}{2} + \cancel{4} \times \frac{\sqrt{3}}{\cancel{2}}$

$= \frac{3}{2} + 2\sqrt{3}$

$= 1.5 + 3.46$

$= 4.96$

New Transformed point is P' is $(2, 0.59, 4.96, 1)$

representing as

$$P' = \begin{bmatrix} 2 \\ 0.598 \\ 4.964 \\ 1 \end{bmatrix}$$

(ii) Shearing in y-direction (3D)

$$Sh_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ sh_{yx} & 1 & sh_{yz} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$y' = sh_{yx} * x + y + sh_{yz} * z$$

Q. Shear the point $P(2,3,4)$ in the y-direction with $sh_{yx} = 1.2$ and $sh_{yz} = -0.7$

$$sh_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1.2 & 1 & -0.7 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

another way

$y' = sh_{yx} * x + y + sh_{yz} * z$

$= 1.2 * 2 + 3 + (-0.7) * 4$

$= 2.4 + 3 \quad - 2.8$

$= 2.6$

Hence, $P' = \begin{bmatrix} 2 \\ 2.6 \\ 4 \\ 1 \end{bmatrix}$

3.3 Successive and Composite Transformations

Successive Transformation

→ Applying multiple transformations (translation, rotation, scaling etc) in a sequence to an object.

→ Order Matters (Sequence): The sequence of transformations affects the final result.

Example: Translating an object and then rotating it will give a different result than rotating it first and then translating it.

→ Non-Commutative :- $T \times R \neq R \times T$ (Translation followed by Rotation ≠ Rotation followed by Translation).

Solution successive
1. Apply $T1 \rightarrow$ Get p'
2. Apply $T2$ on $p' \rightarrow$ Get p''

Composite Transformation

↳ Combining multiple transformations into a single transformation matrix.

↳ Achieved by multiplying individual transformation matrices.

eg $T = T_1 * T_2 * T_3$, where T_1, T_2, T_3 are individual transformation matrices.

$$T_{composite} = T_2 * T_1$$

Then the transformation is applied as

$$p'' = T_{composite} * p$$

Advantages:

Reduces Computation
Improves efficiency in graphics processing.

Q. A point $P(2,3)$ is first translated by $(1,2)$ and then scaled by $(2,3)$. find the final coordinates of the point after both transformations.

Solution

given
$P(2,3)$
$t_x = 1$
$t_y = 2$

$P'(x', y') = ?$

$x' = x + t_x$
$= 2+1$
$= 3$

$y' = y + t_y$
$= 3+2$
$= 5$

$P'(3, 5)$

Now
scaling
$P'(3,5)$
$S_x = 2$
$S_y = 3$

~~$P''(\ )$~~

$P''(x'', y'') = ?$

$S_x = x * S_x$
$= 3*2$
$= 6$

$S_y = y * S_y$
$= 5 \times 3$
$= 15$

$P''(6,15)$

Hence
final coordinates of the point after both transformation are
$P''(6, 15)$

Solution
Compositive transformation matrix

Translation matrix

$$T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Composite matrix

$$T_{Composite} = S \times T$$
$$= \begin{bmatrix} 2 & 0 & 2 \\ 0 & 3 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying the composite matrix to the original point $P(2,3)$.

$$P'' = T_{composite} \times P$$
$$= \begin{bmatrix} 2 & 0 & 2 \\ 0 & 3 & 6 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ 15 \\ 1 \end{bmatrix}$$ So, $P''(6,15)$ Ans

Window to Viewport Transformation

Window to viewport transformation is a technique used in computer graphics to map a portion of a world coordinate scene (called the **window**) onto a display area (called the **viewport**). This process ensures that a selected portion of the scene is properly displayed on the screen.

Concept

1. **Window**: The rectangular area in world coordinates that you want to display.
2. **Viewport**: The rectangular area on the screen where the window is mapped.
3. **Scaling and Translation**: The transformation involves scaling the window dimensions to fit into the viewport and translating it to the correct position on the screen.

Formula for Transformation

To transform a point (X_w, Y_w) from the window to the viewport, we use:

$$X_v = X_{vmin} + \left(\frac{X_w - X_{wmin}}{X_{wmax} - X_{wmin}}\right) \times (X_{vmax} - X_{vmin})$$

$$Y_v = Y_{vmin} + \left(\frac{Y_w - Y_{wmin}}{Y_{wmax} - Y_{wmin}}\right) \times (Y_{vmax} - Y_{vmin})$$

Where,

(X_w, Y_w) is point in the window

(X_v, Y_v) is corresponding point in the viewport

$X_{wmin}, Y_{wmax}, X_{wmin}, Y_{wmax}$ is window boundaries

$X_{vmin}, Y_{vmax}, X_{vmin}, Y_{vmin}$ is Viewport boundaries

Example:

Given: Window: $Xwmin$ = 10 , $Xwmax$ = 50 , $Ywmin$ = 20 , $Ywmax$ = 60 X wmin =10,X wmax =50,Y wmin =20,Y wmax =60 Viewport: $Xvmin$ = 100 , $Xvmax$ = 300 , $Yvmin$ = 200 , $Yvmax$ = 400 Xvmin =100, Xvmax =300,Y vmin =200,Y vmax =400 Point in the window: (Xw , Yw) = (30 , 40) (X w ,Y w)=(30,40)

$$X_v = 100 + \left(\frac{30 - 10}{50 - 10}\right) \times (300 - 100)$$

$$X_v = 100 + \left(\frac{20}{40}\right) \times 200 = 100 + (0.5 \times 200) = 200$$

$$Y_v = 200 + \left(\frac{40 - 20}{60 - 20}\right) \times (400 - 200)$$

$$Y_v = 200 + \left(\frac{20}{40}\right) \times 200 = 200 + (0.5 \times 200) = 300$$

The point (30,40) in the window maps to (200,300) in the viewport.